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ROBERT HERMANN, P.I.

The aim of this research is to develop new mathematical methodology for the analysis of hybrid systems [2] of the type involved in Air Traffic Control (Abb: ATC) problems. Two directions of investigation were initiated. The First used the methodology of Nonlinear Generalized Functions, whose mathematical foundations were initiated by Colombeau [3] and developed further by Oberguggenberger [5]; it has been extended to apply to Ordinary Differential Systems of the type encountered in Control in joint work [1] with the PI and M. Oberguggenberger of U. of Innsbruck. This involved a 'mixture' of 'continuous' and discrete' methodology. (Unfortunately, only preliminary work was done in either direction, and the Grant has not been renewed!) A joint paper has been written [1] with Oberguggenberger and will be published by Pittman Publishers as part of a collection resulting from a Workshop held in Vienna in October, 1997. The Second direction involved utilization of the purely 'discrete' methodology of Cellular Automata (Abb: CA) Theory.

ATC clearly involves mixtures of two sorts of mathematical Problems: The 'Continuous' Dynamics of a standard control type described by ordinary differential equations (Abb: ODE) of the form: $\{dx/dt = f(x, u)\}$ and the Discrete Lattice Dynamics involved of cellular automata. Most of the CA literature [6-9] involves a discretization of a partial differential equation system of the type encountered in physics problems (e.g. fluid and gas problems.) Both of these directions requires much thinking and new development of mathematical fundamentals before they may be utilized in the ATC work. (For example, I had to spend a considerable amount of time learning the CA literature [5-9], and have not yet been able to do prepare anything for publication in this area.) Rather than consider CA as 'discretization' of PDE systems, I believe that the ATC applications will require a completely different and new mathematical methodology, a sort of discrete analogue of Jet Bundles and/or the Sheaf-theoretic techniques of topologists. Here too, I have begun work on virtually 'virgin' mathematical ground (at least from an 'applied' point of view) which will require considerable preliminary work.

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One starting point for the Generalized Function approach is the standard control equations $\{dx/dt = f(x, u)\}$. Usually in the Control literature 'f' and 'u' are functions with enough 'smoothness' so that standard ODE methodology applies. In Hybrid Control situations [2] we are interested in more 'pathological' behavior, e.g. 'delta functions'. Colombeau's methodology [3] seems best adapted to utilization of the Lie-theoretic methodology that has been so fruitful [4] in the standard 'smooth' control situations. Oberguggenberger and I have initiated [1] development of the required analytical and algebraic methodology. The next step should be to extend these Lie algebra techniques to the 'generalized' case : e.g. a 'Caratheodory - Chow Theorem in the Generalized framework. While I have some preliminary results in this direction (which I hope to write about in a successor paper to [1] with Oberguggenberger) without further support from NASA I will not be able to work as diligently in this area as I had planned. I have applied for a Guggenheim Fellowship to write one or more books (with M. Hazewinkel as co-author) fleshing out the material in my "Interdisciplinary Mathematics" books. Some of these applications of Generalized Function Theory to Control problems will be developed there if I am successful with Guggenheim, or can find another source of funding.

In any case, my work has been supported for twenty years by NASA, for which I am very grateful!

Bibliography

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